Astronomical Approximations. II, III. By Pliny Earle Chase, LL.D., Professor of Philosophy in Haverford College.

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II. Velocity of Light, and Kirkwood's Analogy.

The cosmical undulations should produce effects at every centre of inertial reaction, which would furnish data for approximate determinations of the velocity of light. We have seen that the favorable central position of the Earth, in the belt of greatest condensation, leads to a simple equation for Sun's apparent diameter and, therefore, for finding the quotient of Earth's distance from Sun by Sun's semi-diameter. The accuracy of the result is confirmed by other inferences which may be drawn from the same data.

Kirkwood's Analogy may be formulated thus:

$$\left(\frac{\rho_n}{\rho_0}\right)^3 \times \left(\frac{\nu_0}{\nu_n}\right)^2 = \text{a constant}\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

Let ρ_0 denote Sun's semi-diameter; ρ_1, ρ_2, etc., the mean vector-radii of the several planets (Mercury$_1$, Venus$_2$, etc.); μ, mass; t, time of rotation synchronous with revolution at Laplace's limit; ν_n, number of rotations in ν_0 orbital revolutions synchronous with primitive solar rotation; v_λ, velocity of light; v_n, velocity of revolution $(\sqrt{gr})$ at the surface of planet$_n$; r, planetary radius; the subscript figures being applicable to μ, t, ν, v, and r. The actions and reactions of light-waves, between the nucleal centre (Sun) and the principal centre of primitive condensation (Earth), lead to the equation, similar to Kirkwood's:

$$\left(\frac{\rho_n}{\rho_0}\right)^3 \times \left(\frac{\nu_0}{\nu_n}\right)^2 = \left(\frac{v_n}{v_\lambda}\right)^2 \frac{\mu_0^2}{\mu_n\,\mu_3} \times \left(\frac{t_n}{t_3}\right)^2 \frac{r_n}{r_3}\ldots\ldots\ldots(2.)$$

For Earth, $\rho_n = \rho_3 = 214.54\rho_0$; $\nu_n = \nu_3 = 366.256\nu_0$; $v_n = v_3 = .0012383r_3$; $v_\lambda = 214.54\rho_0 \div 497.83 = .43096\rho_0$. Substituting, and taking the square root of equation (2), we get:

$$\frac{214.54^{\frac{3}{2}}}{366.256} = \frac{.0012383r_3}{.43096\rho_0} \times \frac{\mu_0}{\mu_3}\ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

If we designate density by δ, mass varies as $r^3\delta$, or as the square of $\frac{r}{v}$. Therefore $\frac{\delta_3}{\delta_0} = \left(\frac{r_0}{g_0} \div \frac{r_3}{g_3}\right) = \left(\frac{1\text{ year}}{214.54^{\frac{3}{2}}} \div 5074\ sec.\right)^2 = 3.9175.$

Substituting in (3), dividing and reducing:

$$\left.\begin{array}{l} \dfrac{214.54^{\frac{3}{2}}}{366.256} \times \dfrac{.43096}{.0012383} \times 3.9175 = \left(\dfrac{\rho_0}{r_3}\right)^2 \\ 108.155 r_3 = \rho_0 = 428{,}600 \text{ miles.} \\ \rho_3 = 214.54 \rho_0 = 91{,}950{,}000 \text{ miles.} \\ v_\lambda = \rho_3 \div 497.83 = 184{,}710 \text{ miles.} \end{array}\right\} \ldots\ldots\ldots\ldots (4.)$$

If we suppose Sun to contract till Laplace's limit would correspond with Sun's present equatorial radius, the foregoing equations would all be deducible from the following:

$$\left.\begin{array}{l} \dfrac{\mu_3\, t_3}{\mu_0\, t_0} = \dfrac{v_3}{v_\lambda} \\ \dfrac{\mu_3}{\mu_0} \times \dfrac{1 \text{ dy}}{.11624 \text{ dy}} = \dfrac{4.907}{184{,}710} \\ \mu_0 = 323{,}350 \mu_3 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

In these first approximations no allowance has been made for orbital eccentricities, or for disturbances by the principal planets. I am, therefore, inclined to attach more importance to the following methods.

The equivalence of luminous action and reaction, between the nucleal centre (c_0) and the principal centre of primitive condensation (c_3), is shown by Earth's still retaining one-half of the original rupturing force. According to Stockwell, Earth's mean eccentricity is .0338676. If the rupturing locus is represented by mean perihelion, since the superficial velocity of rotation in a condensing nebula varies inversely as radius, the rupturing velocity was $\frac{1}{.9661324}$ times the mean velocity. The constant solar equation $\frac{g_0 t_0}{2} = v_\lambda$, would be satisfied in .9661324 $\times \frac{1}{2}$ yr., if we look only to solar gravity at the corresponding nucleal surface, or in .9661324 yr., if we look to initial terrestrial gravity as one-half of corresponding solar gravity.

$$\left.\begin{array}{l} v_\lambda = .9661324 \times 365.256 \times 86400 \times \dfrac{32.0874}{5280} = 185{,}287 \text{ miles.} \\ \rho_3 = 497.83 v_\lambda = 92{,}242{,}000 \text{ miles.} \end{array}\right\} (6.)$$

In equation (4), if we substitute Earth's mean solar day for the sidereal day, we get:

$$\rho_0 = \left(\frac{366.256}{365.256}\right)^{\frac{1}{2}} \times 428{,}600 = 429{,}200 \text{ miles.}$$

$$\left.\begin{array}{l} \rho_3 = 92{,}070{,}000 \text{ miles.} \\ v_\lambda = 184{,}970 \text{ miles.} \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

By the well-known laws of elasticity, M, the solar *modulus* of light, or the height of a homogeneous æthereal atmosphere, at Sun's surface, which

would transmit undulations with the velocity of light, is $\left(\frac{v_\lambda}{v_0}\right)^2 \rho_0$. This is equivalent to $\left(\frac{.43096}{.00062563}\right)^2 \rho_0 = 474{,}500\rho_0$. For v_λ, as we have already seen, is $.43096\rho_0$, and v_0 is $\frac{2\pi\rho_3 \times (214.54)^{\frac{1}{2}}}{365.256 \times 86400} \times \frac{214.54\rho_0}{\rho_3} = .00062563\rho_0$. If the cyclical variations of alternately increasing and diminishing stress, to which every particle of the Sun is exposed during each half-rotation, are due to the velocity of light, the equations, $\frac{g_0 t_0}{2} = v_\lambda$, and $\sqrt{g_0 \rho_0} = .0006$-$2563\rho_0$, give :

$$\left.\begin{aligned} g_0 &= .00000039142\rho_0 \\ t_0 &= \frac{2 \times .43096\rho_0}{g_0} = 2{,}202{,}050 \text{ sec.} = 25.486 \text{ dy.} \end{aligned}\right\} \quad \ldots\ldots(8.)$$

The continual disturbances at Sun's surface, and the combined influences to rotation and revolution upon spots near the solar equator, make it impossible to find the exact value of t_0 by direct observation. Laplace's estimate was $25\frac{1}{2}$ days; Carrington's 25.38 days. According to his observations, "near the equator the period was about 25.3 days, while it was a day longer in 30° latitude. Moreover, the period of rotation seems to be different at different times, and to vary with the frequency of the spots. But the laws of these variations are not yet established. In consequence of their existence, we cannot fix any definite time of rotation for the Sun, as we can for the Earth and for some of the planets. It varies at different times, and under different circumstances, from 25 to $26\frac{1}{2}$ days." *

It is, therefore, impossible now to assign any more probable value to t_0 than the one which I have deduced theoretically from the stress of luminous waves. If future observations should lead to the acceptance of a period which is either slightly less or slightly greater, the discrepancy can be easily accounted for, either by orbital acceleration or by inertial resistance and retardation.

If ρ_a represents Stockwell's determination of the centre of the belt of greatest condensation ($1.016878\rho_3 = 218.16\rho_0$) and if we suppose a similarity of action and reaction at the nucleal centre (Sun's centre $= c_0$) and at the dense-belt centre (c_a), we find :

$$\left.\begin{aligned} &\left(\frac{M}{\rho_0}\right)^{\frac{1}{2}} \times \left(\frac{\rho_a}{\rho_0}\right)^{\frac{1}{2}} = \frac{1\ yr.}{1\ dy.} \times \frac{g_0}{g_3} \\ &(474500 \times 1.016878 \times 214.54)^{\frac{1}{2}} = 366.256\frac{g_0}{g_3} \\ &g_0 = 27.78 g_3. \\ &\rho_0 = \frac{g_0\, \delta_3}{g_3\, \delta_0} r_3 = 27.783 \times .9175 \times 3962.8 = 431{,}250 \text{ miles.} \\ &\rho_3 = 214.54\rho_0 = 92{,}520{,}000 \text{ miles.} \\ &v_\lambda = 185{,}850 \text{ miles.} \end{aligned}\right\} \quad \ldots.(9.)$$

*Newcomb: *Popular Astronomy*, p. 250.

The influence of luminous undulation is also shown by the principal planet of the light belt (Jupiter), which is also the controlling planet of the system. For the time required by light, to traverse the linear orbit $(4\rho_5)$ which would be synchronous with Jupiter's orbit, is equivalent to the time of satellite revolution at Jupiter's surface $\left(2\pi\sqrt{\frac{r_5}{g_5}}\right)$.

$$\left.\begin{array}{l}
4 \times 5.2028 \times 497.829 = 10360.24 = 2\pi\sqrt{\frac{r_5}{g_5}} \\
\frac{\delta_0}{\delta_5} = \left(\frac{10360}{10043}\right)^2 = 1.0642 \\
\frac{r_0}{r_5} = (1047.875 \div 1.0642)^{\frac{1}{3}} = 9.9485 \\
\frac{g_0}{g_5} = 1047.875 \div (9.9485)^2 = 10.587. \\
\frac{\nu_5}{\nu_0} = 11.86 \times 365.256 \times 24^h \div 9^h\,55^m\,26^s.5^* = 10477.56. \\
\frac{v_\lambda}{v_5} = (10.587 \times 9.9485)^{\frac{1}{2}} \times 688.84 = 7069.5 \\
\frac{t_3}{t_5} = 24^h \div 9^h\,55^m\,26^s.5 = 2.4183. \\
\text{Substituting in (2); } (5.2028 \times 214.54)^3 \div (10477.56)^2 = \frac{1047.88}{7069.5^2} \times \\
\qquad \left(\frac{1}{2.4183}\right)^2 \times \frac{\mu_0\, r_5}{\mu_3\, r_3} \\
\text{Multiplying by } \frac{r_0}{r_5} = 9.9485, \text{ and reducing; } 137718750 = \left(\frac{r_0}{r_3}\right)^4 \\
\rho_0 = 108.33 r_3 = 429{,}300 \text{ miles.} \\
\rho_3 = 214.54 \rho_0 = 92{,}100{,}000 \text{ miles.} \\
v_\lambda = .43096 \rho_0 = 185{,}000 \text{ miles.}
\end{array}\right\} \cdot(10.)$$

The experiments which are now in progress, for measuring the velocity of light, may lend interest to the following comparative tabulation, in kilometers, of some of the most important approximations to the velocity.

	By Measurement.		
Maxwell	(Electricity),	288,000	kil.
Ayrton and Perry	"	298,000	"
Foucault,		298,360	"
Michelson,		299,820	"
Cornu,		300,000	"
	By the Nebular Hypothesis.		
From Kirkwood's Analogy,	(4)	297,254	kil.
" " "	(7)	297,672	"
" Jupiter's density,	(10)	297,720	"
" Earth's mean perihelion	(6)	298,182	"
" Primitive condensation	(9)	299,088	"

*This is the time of Jupiter's rotation, as given by Professor Asaph Hall.

All the elements for the foregoing calculations can be measured with much greater accuracy than the solar parallax, the position of the moon's centre, cometary disturbance, or any other similar astronomical data. The identification of luminous and electro-magnetic action, by Weber, Kohlrausch, Thomson, Maxwell, and Perry and Ayrton, together with Peirce's investigations of the influence of repulsive force in the miniature world-building of cometary nuclei,* lead me to hope that further research will show what modifications are needed in order to secure exact astronomical measurements, by means of the equal action and reaction of opposing forces.

III. Controlling Centres.

The principal centre of gravity in the solar system (Jupiter–Sun), is at $5.2028 \times 214.54\rho_0 \div 1047.88 = 1.06522\rho_0$. The ratio of synchronous lineal and circular orbits $= \frac{2}{\pi}$. The wave-velocity which counteracts Earth's semi-diurnal variations of stress, is $v_\beta = \frac{32.08 \times 43082}{5280} = 261.76$ miles. Equating radial (numerator) and tangential (denominator) influences, we find:

$$\left.\begin{aligned} \frac{v_\lambda}{v_\beta} &= \frac{2}{\pi} \times \frac{1.065\rho_0}{\rho_5} \\ v_\lambda &= 186{,}025 \text{ miles.} \\ \rho_3 &= 92{,}606{,}000 \text{ miles.} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

At any given distance from cosmical centres the orbital influence is proportioned to the mass. Hence the equation:

$$\left.\begin{aligned} &\frac{v_\lambda}{v_\beta} \times \frac{v_5}{v_3} = \frac{\mu_5}{\mu_3} \\ &\frac{186{,}025}{261.76} \times \sqrt{\frac{1}{5.2028}} = 311.56 \\ &\frac{\mu_0}{\mu_3} = 311.56 \times 1047.88 = 326{,}500 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (2.)$$

A similar reciprocity, introducing some further interesting considerations, may be found by looking to the centre of reciprocal nebular rupture, Neptune's secular perihelion. Adopting Stockwell's value of Neptune's greatest eccentricity (.0145066), and taking the mean between Stockwell's (30.03386) and Newcomb's (30.05437) estimates of Neptune's mean radius-vector, Neptune's secular perihelion (ω) is at $3\pi^2\rho_3$. Both the linear centre of oscillation and the collisions of subsiding particles† tend to produce cosmi-

*Trans. Amer. Acad., 1859.

†*Ante*, xvii, 100.

cal aggregations at $\frac{2}{3}r$. This tendency, considering ω as a centre, would fix the boundary of the belt of retrogradely rotating planets at $\frac{1}{3}\ \omega = \pi^2 \rho_3 =$ $9.8695 \rho_3$, or between Saturn's mean and aphelion positions, so that Saturn well represents the surface of the belt of directly rotating planets. When the rotating wave-velocity (w) was operating in Saturn's orbit $\left(\text{at } \frac{\omega}{3}\right)$ the orbital velocity $\left(\frac{w}{\pi}\right)$ was found at $\left(\frac{\omega}{3\pi} = \pi\right)\rho_3$, or in the asteroidal belt (3.142), nearly midway between the mean perihelion of Mars (1.403), and the secular perihelion of Jupiter (4.886), and also nearly midway between Earth's secular aphelion (1.068), and Jupiter's mean distance (5.203), as well as between the mean aphelia of Venus (.774), and Jupiter (5.519). The next change of wave-rotating to orbital velocity brings us to Earth, the central and greatest mass in the belt of greatest condensation. If we start from 2ω, the surface of early subsidence which would give orbital velocity at ω, all these relations may be embodied in the equation :

$$\left.\begin{aligned}
&\left(\frac{v_{\lambda}\, w_3}{g_0 \sqrt{r_0\, r_3}}\right)^2 = 6\ \pi^2 \\
&\left(\frac{688.84}{16.982}\right)^2 \times \frac{g_3}{g_0} = 59.217 \\
&27.785 g_3 = g_0 \\
&\text{By Eq. II., (9); } \rho_3 = 92{,}540{,}000 \text{ miles.} \\
&\mu_0 = 329{,}200 \mu_3
\end{aligned}\right\} \quad \ldots\ldots\ldots(3.)$$

The action and reaction between the reciprocal centre (Neptune) and the centre of condensation (Earth), are also shown in the ratio between v_3 and the velocity of terrestrial rotation (w_3):

$$\left.\begin{aligned}
&\frac{v_3}{w_3} = \frac{4.907}{.289} = 16.982 \\
&\frac{\mu_3}{\mu_8} = \frac{w_3}{v_3} \\
&\frac{\mu_0}{\mu_8} = \frac{329{,}200}{16.982} = 19{,}385
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

Newcomb's estimate for $\frac{\mu_0}{\mu_8}$, as deduced from observations on Neptune's satellite, is 19,380 ± 70. By combining (4) with Eq. (11) in "Further Confirmations of Prediction," * we find the equation between moments of reciprocal rotation $\left(\frac{\mu}{\rho^2}\right)$ and times of synchronous rotation and revolution $\left(2\ \pi \sqrt{\frac{r}{g}}\right)$:

* Ib. xviii, 231.

$$\left.\begin{array}{l} \dfrac{\mu^3\ \rho_8^{\ 2}}{\mu_8\ \rho_3^{\ 2}} = \dfrac{\mu_0\ t_3}{\mu_3\ t_0} \\ \dfrac{19{,}385\rho_8^{\ 2}}{329{,}200\rho_3^{\ 2}} = \dfrac{329200 \times 5074 \text{ sec.}}{1 \times 31558150 \text{ sec.}} \\ \rho_8 = 29.9936\rho_3 \end{array}\right\} \ldots\ldots\ldots\ldots (5.)$$

The Saturnian orbit embraces the primitive centre of rotating inertia; Saturn's mean position having been influenced by the locus of reciprocal rupture (Neptune's *m. p.**), the two chief points of incipient condensation (Jupiter's *s. a.* and Saturn's *s. a.*), and the mean positions of the other planets, as will be seen by the following table, in which Sun's mass = 10,000,000,000:

	μ_n	Authority.	$\rho_n \div \rho_3$	Authority.	$\mu\,\rho^2$
1. Mercury,	2,055	Encke,	.3871 *m.*	Leverrier,	308
2. Venus,	23,406	Hill,	.7233 *m.*	Leverrier,	12,246
3. Earth,	30,600	Newcomb,	1.0000 *m.*	Leverrier,	30,600
4. Mars,	3,233	Hall,	1.5237 *m.*	Leverrier,	7,506
5. Jupiter,	9,543,087	Bessel,	5.5193 *s. a.*	Stockwell,	290,693,300
6. Saturn,	2,855,837	Bessel,	10.3433 *s. a.*	Stockwell,	305,528,600
7. Uranus,	442,478	Newcomb,	19.1834 *m.*	Newcomb,	162,837,000
8. Neptune,	515,996	Newcomb,	29.7322 *m. p.*	Stockwell,	456,140.000

$$\left.\begin{array}{l} \sqrt{\Sigma\mu\rho^2 \div \Sigma\mu} = \rho_6 \\ \Sigma\mu = 13{,}416{,}692 \\ \Sigma\mu\rho^2 = 1{,}215{,}247{,}560 \\ \sqrt{\Sigma\mu\rho^2 \div \Sigma\mu} = 9.517 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

Saturn's mean radius-vector is $9.539\rho_3$. The above result, therefore, indicates a slight preponderance, beyond the orbit of Neptune, of the unknown cosmical matter in our system. If the influence of all this unknown preponderance is equivalent to that of a mass about $\frac{3}{4}$ as great as Earth, at the locus of incipient subsidence ($2\omega = 6\pi^2\rho_3$), the mean moment of nebular rotation of each planet is represented by Saturn's mean position.

μ	$m(\rho_n \div \rho_3)$	$\mu\rho^2$
1	.3871	308
2	.7233	12,246
3	1.0000	30,600
4	1.5237	7,506
5	5.2028	258,317,647
6	9.5388	259,852,941
7	19.1834	162,837,000
8	30.0339	465,444,000
x	59.2170	76,446,000

* *a*, aphelion; *p*, perihelion; *m*, mean; *s*, secular.

$$\left.\begin{aligned} \Sigma\mu &= 13{,}440{,}100 \\ \Sigma\mu\rho^2 &= 1{,}222{,}948{,}248 \\ \sqrt{\Sigma\mu\rho^2 \div \Sigma\mu} &= 9.539 \end{aligned}\right\} \dots\dots\dots\dots(7.)$$

The ratio of Uranus to Neptune appears to have been determined by the incipient condensation of the system. For orbital velocity is proportioned to $\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}$; therefore, for any constant initiatory velocity, like v_{λ}, mass is proportioned to the radius of equal orbital velocity, or inversely to the $\frac{2}{3}$ power of the velocity of reciprocal orbital revolution, or to the cube root of the distance from the Sun. Designating the locus of incipient condensation (Neptune's secular aphelion) by ρ_{γ}, we find

$$\left.\begin{aligned} \mu_7(\rho_{\gamma})^{\frac{1}{3}} &= \mu_8(\rho_7)^{\frac{1}{3}} \\ \mu_7(30.4696)^{\frac{1}{3}} &= \mu_0\frac{(19.1834)^{\frac{1}{3}}}{19385} \\ \frac{\mu_0}{\mu_7} &= 22618. \end{aligned}\right\} \dots\dots\dots\dots(8.)$$

Newcomb's estimate of $\frac{\mu_0}{\mu_7}$ is 22600 ± 100.

The inner retrogradely-rotating planet (Uranus) is connected with the belt of directly-rotating planets by the two proportions :

$$\left.\begin{aligned} &\mu_7 : \mu_5 :: \rho_{3(2)} : \rho_{7(4)} \\ &\mu_7 : \frac{\mu_0}{1047.88} :: .9661 : 20.044 \\ &\frac{\mu_0}{\mu_7} = \frac{20.044 \times 1047.88}{.9661} = 22530 \end{aligned}\right\} \dots\dots\dots\dots(9.)$$

$$\left.\begin{aligned} &\mu_7 : \mu_3 :: v_{0(a)} : v_{0(3)} \\ &\frac{\mu_0}{22530} : \mu_3 :: \sqrt{214.54} : \sqrt{1.019256} \\ &\frac{\mu_0}{\mu_3} = \sqrt{\frac{214.54}{1.019256}} \times 22530 = 326{,}900 \end{aligned}\right\} \dots\dots\dots\dots(10.)$$

In equation (9), $\rho_{3(2)}$ = Earth's mean perihelion ; $\rho_{7(4)}$ = mean aphelion of Uranus. In equation (10), $v_{0(a)}$ = velocity of projection at the mean perihelion centre of gravity of Sun and Jupiter (5.2028 × 214.54 × .95684 ÷ 1047.88 = 1.019256); $v_{0(3)}$ = Earth's mean orbital velocity. The influence of Jupiter's mean perihelion position will be further shown in the following comparisons (13, 14).

In the early ellipsoidal or truncately paraboloidal nucleus indicated by Peirce's cometary and meteoric researches, of which Uranus (19.1836)

represents the perihelion, and Neptune (30.034) represents the aphelion, Jupiter's mean aphelion (5.4274) was central.

The centre of reciprocal rupture (Neptune's secular aphelion = 30.47), the paraboloidal centre (Jupiter's secular aphelion = 5.52), and the centre of the dense belt (Earth = 1), are connected by the geometrical proportion

$$1 : 5.52 : : 5.52 : 30.47 \qquad \text{(11.)}$$

The masses at the centres of rotary inertia (Saturn), and of early nebulosity (Jupiter), are proportioned to their respective gravitating tendencies towards the nucleal centre (Sun), or inversely proportioned to the squares of their vector-radii, so that their primitive moments of rotary inertia were equal.

$$\left.\begin{array}{l} \mu_5 \rho_5^2 = \mu_6 \rho_6^2 \\ \dfrac{5.2028^2 \mu_0}{1047.88} = \mu_6 \times 9.5388^2 \\ \dfrac{\mu_0}{\mu_6} = 3522.3 \end{array}\right\} \qquad \text{(12.)}$$

Bessel's value is 3501.6, so that the theoretical mass is about .006 too small. This approximation, which was first pointed out by Professor Stephen Alexander, convinced me that all the cosmical masses must be determined by ascertainable laws, and thus led me to the results which are embodied in the present and previous communications.

The ratio between the masses at the centre of rotary inertia (Saturn), and at the centre of greatest condensation (Earth), appears to have been determined by Jupiter's perihelion influence and by centrifugal force, since the masses vary nearly inversely as their gravitating tendencies towards the Sun, or directly as the squares of their vector radii.

$$\left.\begin{array}{l} \dfrac{\mu_6 \rho_3^2}{\rho_a} = \dfrac{\mu_3 \rho_6^2}{\rho_0} \\ \dfrac{\mu_0}{3522.3 \times 1.019256} = \mu_3 \times 9.5388^2 \\ \dfrac{\mu_0}{\mu_3} = 326{,}661 \end{array}\right\} \qquad \text{(13.)}$$

The ratio between the masses at the nucleal centre (Sun), and at the centre of primitive nebulosity (Jupiter), combines the projectile, the centrifugal, and the square of the centripetal ratios, thus illustrating the thermodynamic law that equal quantities of heat correspond to equal increments of *vis viva* in simple gases.

$$\left.\begin{array}{l} \dfrac{\mu_0}{\mu_5} = \left(\dfrac{\mu_5}{\mu_6}\right)^2 \times \dfrac{\mu_5}{\mu_3} = \left(\dfrac{\rho_6}{\rho_5}\right)^4 \left(\dfrac{\rho_6}{\rho_3}\right)^2 \times \dfrac{\rho_a}{\rho_0} \\ \dfrac{\mu_0}{\mu_5} = \dfrac{9.5388^6 \times 1.019256}{5.2028^4} \end{array}\right\} \qquad \text{(14.)}$$

The centrifugal ratios between Saturn and Earth (13), and the centripetal ratios between Saturn and Jupiter (12), are further illustrated by the

vector-radii of the centre of inertia and the centre of nebulosity. For, if we take a locus at $\frac{9}{11}$ of ρ_6, ρ_5 is at $\frac{2}{3}$ of the locus, or at the centre of subsidence-collision and the centre of linear oscillation, while the locus itself is at the centre of projection due to Saturn's spherical *vis viva* $\left(.4 \text{ of } \frac{5}{11} = \frac{2}{11}\right)$.

$$\frac{6}{11} \text{ of } 9.5388 = 5.20298 \ldots\ldots\ldots\ldots (15.)$$

This approximation gives a value for Jupiter's mean radius-vector which is only about $\frac{1}{290}$ of one per cent. too large.

In the dense belt, the moment of rotary inertia ($\mu\rho^2$) of Mars (7,506) is $\frac{1}{4}$ of Earth's (30,600), while that of Venus (12,246) is .4 of Earth's, thus indicating the influençe of Sun's mean spherical moment of inertia, when expanded to Earth's orbit. The uncertainty with regard to Mercury's mass is too great to warrant any present speculation as to its origin, or its influence on the stability of the system.

The principal considerations, involved in these approximations, are :

1. Fouıier's theorem, that every periodic vibratory motion can always be regarded as the sum of a certain number of pendulum vibrations.

2. The natural alternation of radial and tangential oscillations, in elastic media surrounding centres of inertia.

3. Maxwell's theorem of equality between *vires vivæ* of translation and *vires vivæ* of rotation.

4. Equality of action and reaction, especially in centripetal and centrifugal tendencies.

5. Perihelion indications of primitive centrifugal or rupturing force, and aphelion indications of primitive centripetal "subsidence."

6. Synchronism of rectilinear ($4r$) and circular ($2\pi r$) orbits.

7. The tendency of nodes in elastic media to establish harmonic nodes.

8. The laws of elasticity which connect arithmetical ratios of distance, with geometric and harmonic ratios of density.

9. The different variability, in condensing nebulæ, of times of rotation ($\propto r^2$) and times of revolution ($\propto r^{\frac{3}{2}}$).

10. Laplace's limitation of rotating elastic stress, by the radius of equal times of rotation and revolution.

11. The counteraction of the cyclical variations of stress, during each half-rotation, by the central force (g), after the analogy of projectiles from the Earth's surface.

12. The constancy, at the nucleal surface of any expanding or contracting nebula, of the stress-opposing value $\frac{gt}{2}$.

13. The tendency, in the primitive rupture of a nebula, to rotations in opposite directions.

14. The continual reciprocal action, between attracting centres, $\left(g \propto \frac{\mu}{d^2}\right)$ of disturbances proportional to mass.

15. The limiting influence of parabolic velocities, upon tendencies to dissociation and to aggregation.

16. The ratio of stress-opposing force, at Laplace's limit, to parabolic $\left(\frac{\pi}{\sqrt{2}}\right)$ and to orbital (π) velocity.

17. The influence of centres of linear and of spherical oscillation.

18. The conjoint influence of centres of nucleation, of density, of nebulosity, of rotary inertia, and of reciprocity.

19. The equations of relation between oscillatory and orbital motion.

20. The interesting and suggestive FACT, important in chemistry and general physics as well as in astronomy, that the central stress-opposing value in the solar system $\left(\frac{gt}{2}\right)$ is the velocity of light.

The Relations of the Crystalline Rocks of Eastern Pennsylvania to the Silurian Limestones and the Hudson River Age of the Hydromice Schists. By Charles E. Hall. With a Plate.

(*Read before the American Philosophical Society, January* 2, 1880.)

Recently Prof. Frazer called the attention of the Academy of Natural Sciences to the fact of the occurrence of the fossil *Buthotrephis flexuosa* in the Peach Bottom roofing slates of York county, Pennsylvania. As Prof. Lesquereux admits that this fossil does not extend below the Trenton limestone, it is in all probability within the Hudson river group. Dr. Emmons assigned this fossil to the Taconic System. Since Dr. Emmons' time, I think the fossiliferous bed of the Taconic system have been pretty well proven to be of the Cambrian series, which would place this Taconic fossil of Emmons somewhere about the Hudson river group.

I embrace this opportunity to state some facts from which I have drawn conclusions concerning the relative positions of the rocks forming the crystalline series of Eastern Pennsylvania.

I shall endeavor to make my statements concise, and I think my reasoning will be understood.

We have the following series of rocks:

First. A series of granitoid, syenitic, quartzose, and micaceous schistose rocks, to be seen on the Delaware river above the city bridge at Trenton, and extending in a south-easterly belt across Bucks and Montgomery counties, as far west as Chestnut Hill, Philadelphia.

Second. A series of syenitic, hornblendic and quartzose rocks extending from the neighborhood of Chestnut Hill westward across the Schuylkill river, and covering a greater part of the northern portion of Delaware county. Fine exposures of this rock are to be seen on the Schuylkill river below Spring Mill, Montgomery county. This series may be the upper members of the first, or that extending from the Delaware river to Chestnut Hill.